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The authors investigate the formation and growth of droplets during throttling of a gas mixture. They take account of coagulation of droplets and also interphase mass transfer. The parameters at the throttle exit are determined, from which one can estimate the droplet growth rate in the tube behind the throttle.

By throttling one can reduce the pressure and temperature of a gas in a low-temperature separation scheme used in industrial gas and gas-condensate deposits in preparing the gas for transporting.

The main element of a throttling equipment is a conical device with cross section reducing along its length. As a rule, it is positioned in a pipeline ahead of a separator intended to separate the liquid phase from the gas.

The gas, a mixture of hydrocarbons with a preponderance of methane (80-90%), moves in the pipeline upstream of the throttle at practically constant pressure and temperature. In the throttle the pressure and temperature drop sharply, which leads to condensation of heavy hydrocarbons and the formation of a finely dispersed fog. Droplets being formed grow vigorously due to condensation and coagulation. The basic problem is to determine the average droplet size and their volume content at the throttle exit.

We shall solve the problem under the following assumptions.

1. The gas is a mixture consisting of two components: the first is light hydrocarbons (methane + ethane), and the second is the heavy hydrocarbons (propane and above). The physical and chemical properties of each component are determined in accordance with [1].

2. When the pressure and temperature are lowered only the second component (a vapor) condenses, while the first component (the gas) remains neutral.

3. The volume content of the condensing liquid phase is small ($w \ll 1$).

4. The rate of formation of the liquid phase volume is determined by the relation [2]

$$I = 6,16\pi\alpha \cdot 10^{19} \frac{\sigma^{7/2} M_1^{1/2} (p^{(2)})^2}{p^3 T^2 S \rho_1} \exp\left\{-\frac{17.6}{\ln^2 S} \left(\frac{M_1}{\rho_1}\right)^2 \left(\frac{\sigma}{T}\right)^3\right\},\tag{1}$$

where $S = p^{(2)}/p_{s}^{(2)}(T)$.

5. Liquid-vapor equilibrium occurs only in the vicinity of a droplet surface, and there-fore the partial vapor pressure at the droplet surface is $p_s^{(2)}$. Far from a droplet there is no equilibrium and the partial pressure is $p^{(2)} = py_2$.

6. The condensation growth of droplets is due to the difference of the partial pressures and therefore to the difference of the molecular concentrations far from the droplet and on the droplet surface, $\Delta y = y_2 - y_{2W}$. In accordance with [3] the flux of condensing vapor towards the droplet is

$$J_{2w} = -\frac{\rho_g D_{21}}{R} \Delta y. \tag{2}$$

7. The distribution of droplets n over volumes v, accounting for condensation growth and coagulation, is the solution of the equation

$$u\frac{\partial n}{\partial x} + \frac{\partial}{\partial v}\left(n\frac{dv}{dt}\right) = I\delta\left(v - v_0\right) + I_{co},$$
(3)

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where

$$I_{\rm co} = \frac{1}{2} \int_0^v K(\omega, v - \omega) n(\omega, x) n(v - \omega, x) d\omega - n(v) \int_0^\infty K(\omega, x) n(\omega) d\omega.$$
(4)

The coagulation kernel K(ω , v) appearing in Eq. (4) is determined by the mechanism of droplet interaction. For turbulent gas flow [4]

$$K(\omega, v) = G(v^{1/3}\omega^{1/6} + v^{1/6}\omega^{1/3}).$$
⁽⁵⁾

8. As follows from [1, 5]:

$$z = (0.41g T_{re} + 0.73)^{Pre} + 0.1p_{re},$$

$$l = \frac{AT_{cr}}{M_1} \{7,08(1 - T_{re})^{0.354} + 10.95\omega(1 - T_{re})^{0.456}\},$$

$$p_s = p_{cr} \left\{ h \left(1 - \frac{T_{cr}}{T} \right) \right\}, \quad h = \frac{\ln p_{cr}}{T_b / T_{cr} - 1}.$$
(6)

The gas specific heat c_p and the surface tension coefficient can be determined with the help of methods described in [1, 6].

The system of equations describing the motion of the vapor mixture in the throttle whose cross sectional area varies along its length according to a given law, F(x), allowing for the processes of vapor condensation and droplet coagulation, consists of the one-dimensional gasdynamic equations [7]

$$\frac{d}{dx}\left(\rho_{g}\mu F\right)=0,\tag{7}$$

$$\rho_{g}u\frac{du}{dx} = -\frac{dp}{dx}, \qquad (8)$$

$$\rho_{g}c_{p}u\frac{dT}{dx} = u\frac{dp}{dx} + \varepsilon, \tag{9}$$

the equations of state of the vapor mixture

$$p = \frac{Az\rho_{\rm g}T}{M_{\rm g}} , \qquad (10)$$

the equation of droplet coagulation, Eq. (3), where the rate of condensation growth of the droplets, allowing for Eq. (2), has the form

$$\frac{dv}{dt} = -\frac{4\pi R^2}{\rho_1} J_{2w},\tag{11}$$

and also the equations describing the variation of the molar concentration of the vapor y_2 :

$$u \frac{d}{dx} (y_2 \rho_g) = -I v_0 \rho_1 + 4\pi \int_0^\infty R^2 J_{2w} n(v, x) dv.$$
(12)

The specific condensation energy appearing in Eq. (9) is

$$\varepsilon = v_0 l \rho_1 I + 4\pi M_1 l \int_0^\infty R^2 J_{2w} n(v, x) dv.$$
 (13)

To simplify the system of equations obtained we introduce the distribution moments

$$m_i = \int_0^\infty v^i n(v, x) \, dv. \tag{14}$$

We shall restrict attention to the first two moments. Then, in accordance with [4], in lieu of Eq. (3) we obtain:

$$u \frac{dm_0}{dx} = I - Gm_0^{3/2} m_1^{1/2}, \qquad u \frac{dm_1}{dx} = Iv_0 + V\overline{3}(4\pi)^{2/3} \frac{\rho_g}{\rho_1} D_{21} \Delta y m_0^{2/3} m_1^{1/3}.$$
(15)

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We note that m_0 and m_1 are equal to the number and volume concentrations of droplets, respectively.

We shall solve the system of equations (7)-(13) and (15), taking account of Eqs. (1), (2) and (6), for given conditions at the throttle inlet: $\rho_g(0) = \rho_0$, $u(0) = u_0$, $m_0(0) = m_1(0) = 0$, $T(0) = T_0$, $p(0) = p_0$, $\Delta y(0) = \Delta y_0$.

We introduce the dimensionless variables

$$p = \frac{p}{p_0}, \quad T = \frac{T}{T_0}, \quad \rho = \frac{\rho_g}{\rho_0}, \quad u = \frac{u}{u_0}, \quad F = \frac{F}{F_0}, \quad x = \frac{x}{L},$$

$$\sigma = \frac{\sigma}{\sigma_0}, \quad m_0 = m_0 v_0(0), \quad v = \frac{v}{v_0(0)}, \quad I = \frac{I}{I_0}, \quad z = \frac{z}{z_0},$$

$$p_{cr} = \frac{p_{cr}}{p_0}, \quad T_{cr} = \frac{T_{cr}}{T_0}, \quad \gamma = \frac{u_0^2}{c_p T_0}, \quad \delta = \frac{LI_0}{\rho_0 u_0 c_p T_0},$$

$$\lambda = \frac{\rho_0 u_0^2}{\rho_0}, \quad e = \left(\frac{3}{4\pi}\right)^{1/3} \frac{4\pi I L D_{21} M_1}{c_p u_0 T_0 v_0^{2/3}(0)}, \quad a = \frac{I_0 L v_0(0)}{u_0},$$

$$\beta = 17, 6 \left(\frac{M_1}{\rho_1}\right)^2 \left(\frac{\sigma_0}{T_0}\right)^3, \quad b = \frac{G_0 I}{u_0 v_0^{1/2}(0)},$$

$$f = a \frac{\rho_1}{\rho_0}, \quad d = 4\pi \left(\frac{3}{4\pi}\right)^{1/3} \frac{L D_{21} v_0^{2/3}(0)}{u_0}, \quad c = d \frac{\rho_0}{\rho_1},$$

$$G_0 = 16 \left(\frac{\Gamma}{3\pi \rho_0 \lambda_0^2(0)}\right)^{1/2}, \quad I_0 v_0(0) = \frac{1.93 \cdot 10^{20} \alpha M_1^{1/2} \sigma_0^{7/2}}{\rho_1 p_0 T_0^2}.$$

In the new variables the system of equations has the form:

$$\frac{d}{dx} (\rho uF) = 0, \quad \frac{dp}{dx} = -\lambda \rho u \frac{du}{dx},$$

$$\rho u \frac{dT}{dx} = \gamma \rho^2 u^2 \frac{du}{dx} + e\rho \Delta y m_0^{2/3} m_1^{1/3} + \delta I,$$

$$u \frac{dm_0}{dx} = a \frac{I}{v_0} - b \frac{m_0^{3/2} m_1^{1/3}}{\rho^{1/2} F^{7/8}},$$

$$u \frac{dm_1}{dx} = aI + c\rho \Delta y m_0^{2/3} m_1^{1/3},$$

$$\rho u \frac{d(\Delta y)}{dx} = -d\Delta y m_0^{2/3} m_1^{1/3} \rho - \frac{d(\rho y_{2w})}{dx} - u\Delta y \frac{d\rho}{dx} - f I v_0, \quad p = z\rho T,$$

$$I = \frac{y_2^2 \sigma^{7/2}}{S \rho T^2} \exp\left(-\beta \frac{\sigma^3}{T^3 \ln^2 S}\right),$$

$$S = 1 + \frac{\Delta y}{y_{2w}}, \quad y_{2w} = \frac{\rho c_T}{p} \exp\left\{h\left(1 - \frac{T c_T}{T}\right)\right\},$$

$$v_0 = \frac{\sigma^3}{p^3}, \quad h = \frac{\ln \rho c_T}{T_b / T_{cT} - 1},$$

$$p(0) = \rho(0) = T(0) = u(0) = 1, \quad \Delta y(0) = \Delta y_0, \quad m_0(0) = m_1(0) = 0.$$
(16)

We have solved Eq. (16) numerically. The variation of throttle cross section with length was given in the form $F = (1 - x/2)^2$.

By way of example we consider subsonic flow of a mixture of methane (90%) and propane (10%). At the throttle inlet we have $T_0 = 293^{\circ}K$; $p_0 = 6-12$ MPa; $u_0 = 5-50$ m/sec; $\rho_{\ell} = 590$ kg/m³; $D_{21} = 10^{-7}$ m²/sec; $\Gamma = 10^{-19}$ J; L = 0.1 m; $F_0 = 0.03$ m²; $S_0 = 1$; $\Delta y_0 = 0$. The basic parameters are calculated using the methods described in [1, 5, 6].



Fig. 1. Variation of pressure along the throttle length for $p_0 = 12$ MN/m²; H = 0.2 m: 1) $u_0 = 50$ m/sec; 2) 30; 3) 10; 4) 5 m/sec.

Fig. 2. Liquid phase content at the throttle exit as a function of the gas speed at the inlet: 1) $p_0 = 12 \text{ MN/m}^2$; 2) 10; 3) 8; 4) 6 MN/m². u_0 , m/sec.

As can be seen from Fig. 1, the main pressure drop occurs in a short section of the throttle near the throat. The gas density and temperature behave analogously. The super-saturation of the mixture increases monotonically along the throttle, and the main variation of S occurs also near the throat. In the same region there begins mass condensation of vapor whose volume content m_1 reaches a noticeable value at the throttle exit (Fig. 2). At the value m_1 the pressure at the inlet has a greater influence than do the gas speed and temperature.

In the throttle the size of the droplets increases (Fig. 3). Their growth is due to two mechanisms: vapor condensation on the droplet surface, and coagulation of droplets. The condensation growth rate is determined mainly by the pressure. The coagulation rate depends on the speed and the volume content w, and this means also the pressure. Calculations show that the droplet growth rate in the throttle arises from the fact that the characteristic time for coagulation is much larger than the characteristic time for condensation, comparable with the dwell time of the mixture in the throttle.

Until now we have considered the case when the second component (the vapor) at the inlet is at constant saturation ($S_0 = 1$ or $\Delta y_0 = 0$). In the case $\Delta y_0 < 0$ the vapor condensation region moves to the throttle exit section (Fig. 4). For further decrease of the value of Δy_0 condensation stops in the throttle.

Downstream of the throttle the gas containing droplets of condensate moves towards the separator pipe with unchanged values of pressure and temperature. The supersaturation of the mixture falls sharply to 1, and further growth of droplets is due to droplet coagulation. Putting $I = \Delta y = 0$ in Eq. (16), we obtain the following expression for the variation of the average volume of droplets:

$$\frac{v}{v_1} = (1 + \beta_1 t)^2, \ \beta_1 = 8w_1 \left(\frac{\Gamma}{3\pi\rho_1 \lambda_{01}^2 v_1}\right)^{1/2},$$
(17)

where v_1 , w_1 , ρ_1 correspond to the values at the throttle exit, and λ_{01} is determined by the pipe parameters.

As a rule, $\beta_1 t >> 1$, and therefore the final droplet size is practically independent of the initial size at the throttle exit.

It is known [8] that the average droplet radius in turbulent gas flow in a pipe is limited to a critical radius R_{cr} , since droplets with $R > R_{cr}$ are very likely to break up. An expression for R_{cr} , obtained from reducing experimental data, is given in [9]:

$$R_{\rm er} = 0.09H \left[\frac{1}{u} \left(\frac{2\sigma}{\rho_1 H} \right)^{1/2} \right]^{6/7} \left(\frac{\rho_{\rm g}}{\rho_1} \right)^{1/7}.$$
 (18)

From Eqs. (17) and (18) we find the distance from the throttle exit section at which the droplets are enlarged to the critical size:

$$L_{\rm b} = \frac{0.016\mu_{\rm g}^{3/4} (2\sigma)^{9/14} H^{31/28}}{\omega \Gamma^{1/2} \rho_1^{6/7} u^{29/28} \rho_{\rm g}^{1/28}} \,. \tag{19}$$



Fig. 3. Average droplet volume at the throttle exit as a function of the inlet pressure: 1) $u_0 = 50 \text{ m/sec}$; 2) 30; 3) 10.

Fig. 4. Influence of the deviation from saturation of the second component of the gas mixture on the volume content of the liquid phase at the throttle exit for $p_0 = 12 \text{ MN/m}^2$, $u_0 = 50 \text{ m/sec: } 1) \text{ Ay}_0 = 0$; 2) -10^{-4} ; 3) $5 \cdot 10^{-4}$; 4) -10^{-3} .

We should position the separatorat this distance from the throttle so that we can remove the droplets from the gas flow with maximum efficiency.

NOTATION

w, volume content of liquid phase; I, rate of droplet formation; v_0 , volume of nucleated droplets; v, droplet volume; σ , surface tension coefficient; α , condensation coefficient; M_{ℓ} , M_g , molecular weight of the liquid and gas phases; ρ_{ℓ} , ρ_g , density of the liquid and gas phases; S, supersaturation of the second component; p, pressure; T, temperature; p_s , saturation pressure; y, molar concentration of the component in the gas phase; D, diffusion coefficient; R, droplet radius; n(v), distribution of droplets by volume; I_{co} , collisional term in the droplet coagulation equation; Γ , Gamaker constant; λ_0 , internal scale of gas turbulence; ℓ , specific heat of vaporization; A, gas constant; T_{cr} , p_{cr} , critical values of temperature and pressure; T_{re} , p_{re} , reduced values of temperature and pressure; ω , acentric factor; T_b , boiling temperature; F, cross-sectional area of throttle; u, gas velocity; c_p , specific heat; m_0 , m_1 , distribution moments; L, characteristic length; H, pipe diameter; z, compressibility factor; μ_g , gas viscosity.

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